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OPTIMIZATION OF LINEAR OBSERVATIONS FOR THE STATIONARY KALMAN FILTER BASED ON A GENERALIZED WATER FILLING THEOREM

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Abstract. *We are concerned with an optimization problem of the gain matrix which is used in the linear observation for the Kalman filter. The innovations process included in the Kalman filter has the same structure as the model of parallel Gaussian channels with output feedback which is commonly used in the optimal transmission problem. It is well-known that the optimal output feedback which minimizes the power of the linearly encoded signal is given by the least-squares estimate of the linear term and that the channel outputs then become the innovations process. The remaining part, determination of the linear coefficients in the coding problem, corresponds to our problem, i.e., the optimal selection of the gain matrix in the observation. Firstly, by applying a solution of the optimal transmission problem for this model, we obtain a set of gain matrices which maximize the mutual information between the observation and the signal under a constraint on the power of the innovations process. Secondly, we select the optimal gain matrix from this set in such a way that the estimation error is minimized.*
Keywords: Gaussian processes, Optimal transmission, Least-squares state estimation, Kalman filter

1. **Introduction.** In connection with the Kalman filter [1], the optimization of observations has been extensively studied [2-5,8,9]. There are mainly two kinds of optimization problems concerned with the observations, i.e., sensor scheduling and sensor allocation problems.

The sensor scheduling problem is the one of finding the optimal sensor switching scheme under the constraint that only one sensor among a number of sensors can be used at any time. Athans [2] first discussed and solved this problem by applying the matrix minimum principle which is also his own work. Logothetis et. al [3] discussed the sensor scheduling problem for linear observations via information theoretic criteria. They also treated a discrete-time linear system, and derived equations for the optimal binary-valued sensor gains which maximize the mutual information between the signal and the observation. Their result is equivalent to that of Athans [2] which was obtained as the optimal sensor gains that minimize the variance of the estimation error.

The other kind of problem is that of sensor allocation. Namely, for a number of sensors which are simultaneously available at one time, we want to determine their gains optimally. This type of problem is often formulated as a kind of optimal control problem with a performance criterion which is a function of the covariance of the estimation error, similar to the sensor scheduling, but with a quadratic term of the sensor gains. However, because of the nonlinearity which exists in the Riccati equation of the error covariance matrix, there are few results which are applicable in the real design of the Kalman filter.