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ON PARAMETER ESTIMATION OF STOCHASTIC VOLATILITY MODELS FROM STOCK DATA USING PARTICLE FILTER -APPLICATION TO AEX INDEX-

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Abstract. *We consider the problem of estimating stochastic volatility from stock data. The estimation of the volatility process of the Heston model is not in the usual framework of the filtering theory. Discretizing the continuous Heston model to the discrete-time one, we can derive the exact volatility filter and realize this filter with the aid of particle filter algorithm. In this paper, we derive the optimal importance function and construct the particle filter algorithm for the discrete-time Heston model. The parameters contained in system model are also estimated by constructing the augmented states for the system and parameters. The developed method is applied to the real data (AEX index).*
Keywords: Stochastic volatility, Heston model, Parameter estimation, Particle filter, AEX index

1. **Introduction.** Due to the apperant contradiction of constant volatility assumption of the Black-Sholes model as illustrated by the volatility skew observed in practice, the stochastic volatility models were proposed and applied to the option pricing problems in [1, 2]. We consider the simple stochastic volatility model proposed by Heston [2]:

$$dS_t = \mu_S S_t dt + \sqrt{v_t} S_t dB_t \tag{1}$$

$$dv_t = \kappa(\theta - v_t)dt + \xi\sqrt{v_t}dZ_t \tag{2}$$

where B_t and Z_t are standard Brownian motion processes with the correlation ρ. From the observed stock price S_t, we construct the transformed observation process $y_t = \log S_t/S_0$;

$$dy_t = (\mu_S - \frac{1}{2}v_t)dt + \sqrt{v_t}dB_t. \tag{3}$$

We are interested in estimating the volatility process v_t, for each fixed t, based on our observation data $\{y_s\}_{0\le s\le t}$. Setting

$$\tilde{Z}_t = \frac{1}{\sqrt{1-\rho^2}}(Z_t - \rho B_t),$$

we find that $\tilde{Z}_t$ is independent of B_t. Noting that

$$\begin{aligned} dZ_t &= \sqrt{1-\rho^2}d\tilde{Z}_t + \rho dB_t \\ &= \sqrt{1-\rho^2}d\tilde{Z}_t + \frac{\rho}{\sqrt{v_t}}(dy_t - (\mu_S - \frac{1}{2}v_t)dt), \end{aligned}$$