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BIFURCATION ANALYSIS OF NOISE-INDUCED SYNCHRONIZATION

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Abstract. *We investigate bifurcation phenomena between slow and fast convergences of synchronization errors arising in the proposed synchronization system consisting of two identical nonlinear dynamical systems linked by a common noisy input only. The numerical continuation of the saddle-node bifurcation set of the primary resonance of moments provides an effective identifier of the slow convergence of synchronization errors.*

Keywords: Noise, Synchronization, Bifurcation, Statistical equivalent approach

1. **Introduction.** The noise-induced synchronization of a dynamical system with its copies can easily be found in nonlinear systems, such as the discrete maps [1, 2], the Lorenz system [1], the Duffing oscillator [3], the single mode CO_2 laser [4], and the uncoupled neurons [5]. One of the most important results of them is that the perfect synchronization may arise under some suitable conditions [1, 3, 4]. Moreover, the perfect synchronization exhibits significant degree of robustness against mismatches among the copies such as the parameters mismatch [3] and the independent random fluctuations of the copies [1, 6]. Furthermore, regarding the response of the synchronization system as a Markov process to derive the transition law of it, we have analytically shown that the perfect synchronization can be regarded as an absorbing barrier of the Markov process [7].

In these studies, however, little research have been done on transient behavior to approach the perfect synchronization. In engineering applications, too slow convergence of synchronization errors would be regarded as failing to converge, even if the synchronization is achieved mathematically. In practice, the perfect synchronization is possibly applicable to synchronizing initial conditions of coexisting oscillators with a common specification such as independent subcircuits in a circuit system, independent mechanical vibrators on bench testing, and so on. However, it is hardly applicable to industrial purposes when the convergence speed is too slow.

To solve this problem, we have already investigated how to characterize the slow convergence of synchronization errors [8, 9], showing that the slow convergence is related to the slow diffusions caused by multimodal probability densities so that it can be detected as multi-valued solutions of the moment differential equations (MDE).

In this paper, we perform nonlinear analysis on the MDE. We first construct Poincaré maps of periodic solutions of the MDE to examine asymptotic behavior of the moments and clarify that the primary resonance encounters a saddle-node bifurcation. We then