A METHOD OF OPTIMIZATION OF LINEAR OBSERVATIONS
FOR THE KALMAN FILTER BASED ON
A GENERALIZED WATER FILLING THEOREM

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Abstract. We are concerned with an optimization problem of the gain matrix, over a
given time interval, of the linear observation for the Kalman filter. The innovations pro-
cess included in the Kalman filter has the same structure as the model of a set of parallel
transmission channels with the optimal output feedback. In the linear coding problem for
this set of channels, it is well-known that the optimal output feedback which minimizes
the power of the encoded signal is given by the least-squares estimate of the linear term in
the coding and that the channel output then becomes the innovations process. By applying
this solution of the optimal transmission problem, we obtain, at any time points, a set
of the gains, with non-unique elements, which maximize the mutual information between
the observation and the signal under a constraint on the power of the innovations pro-
cess. Finally, the gain matrix is uniquely determined by an optimization of minimizing
estimation error which is local in time.

Keywords: Gaussian processes, Kalman filter, Least-squares state estimation, Optimal
transmission

1. Introduction. Starting from 1970’s, the optimization problem of observations associ-
ated with the Kalman filter has been studied in many literatures. Most of them formulated
the problem as a kind of optimal control problem with a quadratic performance criterion.
However, because of the existence of the nonlinearity in the Riccati equation of the esti-
mation error covariance matrix, there are few result which is applicable in the real design
of the Kalman filter. In this paper, we apply the fact that the innovations process is de-
scribed by the same equation which was used in the discussion of the optimal transmission
problem of Gaussian signals through parallel channels with feedback [1-5]. According to
the Shannon’s information theory, the observation is considered to be better when the
mutual information between the signal and the observation takes a larger value. On the
other hand, from the view point of the performance of the Kalman filter, the observation
is better when we have a smaller estimation error. Taking into account of these points,
our approach in this paper is quite new and by the following two steps:

(i) Information theoretic optimization to maximize the mutual information between
the signal and the observation subject to a power constraint concerned with the
innovations process;
(ii) Optimization of the performance of the Kalman filter, i.e., minimization of the esti-
mation error variance.

By using this type of approach, we already discussed the optimization problem for the sta-
tionary Kalman filter [6] where all the coefficients of the signal and observations equations
are time-invariant. In this paper, we are concerned with the optimization of time-varying