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TESTING FOR LÉVY TYPE JUMPS IN JAPANESE STOCK MARKET UNDER THE FINANCIAL CRISIS USING HIGH-FREQUENCY DATA

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Abstract. *In this paper, we tested for Lévy jumps using the Lee-Hannig (LH) method and verified that they exist in Japanese stock market. We specifically focus on and use data in 2008 since the financial crisis occurred then. We also estimate distributions for the sizes of jumps and their frequencies with the LH method.*
Keywords: Testing for jumps, Lévy process, The financial crisis, Japanese stock market

1. **Introduction.** In this paper, we detected the existence of Lévy jumps in Japanese stock market through hypothesis testing. As literature has pointed out, Brownian motion (Gaussian white noise and the Wiener process) is insufficient to describe the randomness of fluctuations in stock prices, e.g., fat-tail problems. Consequently, many researchers and practitioners are beginning to apply jump-diffusion models and Lévy jump models as stock price processes to problems of finance. There have been several empirical results with respect to financial markets, e.g., Akahori and Liu [2], Fujisaki and Zhang [6]. However, there have been few results (if they exist, they would mainly be in U.S. markets) with respect to the existence of jumps in stock markets. To confirm researches concerning jump-diffusion and Lévy jump models are worth while, we attempt to test for jumps, especially in Japanese market, which is one of the most important stock markets in the world.

When we model stock price processes mathematically under Markovian settings, we group them into three types of categories: (i) using only Brownian motion noise, (ii) using Brownian motion noise and Poisson jump noise, and (iii) using Lévy jump noise. Note that Lévy jumps include Brownian motion and Poisson jumps. An important feature of the method of analysis discussed in this paper is that we can consider two causes for extensive changes in stock price processes, i.e., jumps and large volatility values. Volatility is a parameter that characterizes the extent of variations in stock price processes. Hence, we need to test the hypothesis after eliminating the effect of volatility to determine whether jumps exist. Our method of analysis, which is explained below, eradicates their influence by using a kind of "standardization". Therefore, we can distinguish large movements in stock price processes caused by jumps from those caused by high volatility values. Various methods of detecting the existence of Poisson jumps (for (ii)) have been presented by several authors, e.g., the Barndorff-Nielsen and Shephard (BNS) [5], the Jiang and Oomen (JO) [7] and the Lee and Mykland (LM) [9] methods. The empirical results from using these methods have been presented by Tassel [12] for the New York Stock Exchange and by Barada, Kubo and Yasuda [4] for Japanese stock markets. Methods relevant to Lévy jumps (for (iii)) have recently been introduced by Aït-Sahalia and Jacod (AJ) [1] and Lee and Hannig (LH) [8]. The authors also presented a few empirical results obtained with their methods from U.S. stock markets. However, there are no results for Japanese stock markets. These tests for whether jumps exist are not only important to financial fields but also other applications that are related to jump processes, e.g., those described by Qiu et al. [10] and Su et al. [11].

We apply the LH method to Japanese markets since it cannot only detect whether jumps exist but also points in time that had jumps (jump-times). We can also estimate distributions in the sizes of jumps and their frequencies in stock price processes through results obtained from jump times. We study the features of Lévy jumps in Japanese stock markets during the financial crisis that happened in September 2008. Therefore, in order to understand the features of the financial crisis, we use data of the Nikkei225 and individual stocks, which are used to construct the Nikkei225, from April 2008 to March 2009 (referred to as 2008 after this). We also compare these with the results from the next year (April 2009 to March 2010, referred to as 2009 after this).

This paper is organized as follows. Section 2 explains our method of testing for jumps, i.e., the LH method. Section 3 presents empirical results from the Japanese stock market in 2008. We treat the existence of jumps in Section 3.1 and provide estimates of distributions in jump sizes and their frequencies in Section 3.2. Conclusions are presented in Section 4.

2. Lee-Hannig Test for Jumps.

2.1. **Models.** Let $[0, T]$ be a fixed time interval, where $T > 0$ represents maturity. We denote S_t as the asset price at time t. When there are no jumps in the stock price, we assume that log-price $\log S_t$ is

$$d \log S_t = \mu_t dt + \sigma_t dW_t, \tag{1}$$

where W_t is standard Brownian motion drift μ_t and spot volatility σ_t are adapted processes such that the underlying process is diffusion, which only has continuous sample paths. When there are Lévy jumps, we assume that S_t is given by a Lévy jump-diffusion model as

$$d \log S_t = \mu_t dt + \sigma_t dW_t + dL_t, \tag{2}$$

where L_t is an adapted Lévy jump process with Lévy jump measure ν that is independent of W_t. Observation of S_t, or equivalently $\log S_t$, only occurs at discrete times $0 = t_0 < t_1 < \cdots < t_n = T$. Here, we assume that observation times are equally spaced : $\Delta t = t_i - t_{i-1}$ for $i = 1, 2, \cdots, n$. We also impose the following necessary assumption on price processes.

Assumption 1. *Properties of drift μ_t and diffusion σ_t processes: For any $\epsilon > 0$,*

- **(A1.1)** $\sup_{i=0,1,\ldots,n-1} \sup_{t_i \le u \le t_{i+1}} |\mu_u - \mu_{t_i}| = O_p\left(\Delta t^{1/2-\epsilon}\right)$,
- **(A1.2)** $\sup_{i=0,1,\ldots,n-1} \sup_{t_i \le u \le t_{i+1}} |\log \sigma_u - \log \sigma_{t_i}| = O_p\left(\Delta t^{1/2-\epsilon}\right)$.

We use the O_p notation in this study to mean that, if for each $\delta > 0$ for random vectors $\{X_n\}$ and nonnegative random variable $\{d_n\}$, $X_n = O_p(d_n)$, there eventually exists finite constant M_δ such that $P(|X_n| > M_\delta d_n) < \delta$.

2.2. **Method of detecting large jumps.** This section explains the method we used for detecting large jumps, which was reported by Lee and Hannig. We introduce window size $K \in \mathbb{N}$, which is used to estimate volatility values.

Definition 2.1. *(Definition 1 in Lee and Hannig [8]) The piecewise constant process, J_t, which will be used to test whether there was a Lévy jump from t_{i-1} to t_i, is defined for*

$t \in (t_{i-1}, t_i]$ as

$$J_t := \frac{\log(S_{t_i}/S_{t_{i-1}})}{\hat{\sigma}_{t_i}\Delta t^{1/2}},$$

where for any $g > 0$, $0 < \omega < 1/2$, and $t \in (t_{i-1}, t_i]$,

$$\hat{\sigma}_t^2 := \frac{\Delta t^{-1}}{K}\sum_{j=i-K}^{i-1}\left(\log\frac{S_{t_j}}{S_{t_{j-1}}}\right)^2 I_{\{|\log(S_{t_j}/S_{t_{j-1}})|\le g\Delta t^{\omega}\}},$$

where I_A is the indicator function for set A.

Remark 2.1.
(1) *From Proposition 1 in Lee and Hannig* [8], *for the process (2), we have $\hat{\sigma}_\tau \xrightarrow{P} \sigma_\tau$ as $\Delta \to 0$ for any stopping time $\tau > 0$ independent of process S_t under $K \to \infty$, $\Delta t K \to \infty$ and Assumption 1.*
(2) *Lee and Hannig* [8] *proposed that the way to choose the window size was to find an optimal $K = b\Delta^c$, with $-1 < c < 0$ for some constant b. We used this approach in our analysis.*

Theorem 2.1 below describes the limiting behavior of their test statistics.

Theorem 2.1. *(Proposition 2 in Lee and Hannig* [8]*: Large Lévy jump-detection rule). Let J_t be the same as in Definition 2.1 and $K \to \infty$ and $\Delta t K \to 0$. Suppose the process follows (1) and Assumption 1 is satisfied. Then, as $\Delta t \to 0$,*

$$\frac{\max_{t\in(t_{i-1},t_i]\ for\ 0\le i\le n}|J_t| - C_n}{S_n} \xrightarrow{D} \xi,$$

where ξ has cumulative distribution function $P(\xi \le x) = \exp(-e^{-x})$,

$$C_n = (2\log n)^{1/2} - \frac{\log\pi + \log(\log n)}{2(2\log n)^{1/2}} \text{ and } S_n = \frac{1}{(2\log n)^{1/2}}.$$

Remark 2.2. *Under the null hypothesis "there is no jump on $(t_{i-1}, t_i]$", we detect the arrival of large jumps at testing time t_i if the absolute value of the test statistic is bigger than $q_{\tilde{\alpha}}S_n + C_n$, where $q_{\tilde{\alpha}}$ is the $\tilde{\alpha}$ quantile of the limiting distribution of maximum ξ.*

2.3. **Method of detecting small jumps.** Next, we will describe our approach to finding the presence of small jumps.

Theorem 2.2. *(Proposition 3 in Lee and Hannig* [8]*). Let J_t be the same as in Definition 2.1 and $K \to \infty$ and $\Delta t K \to 0$. Suppose the process follows (1) and Assumption 1 is satisfied. Then, as $\Delta t \to 0$,*

$$J_t \xrightarrow{D} N(0,1),$$

where $N(0,1)$ denotes a standard normal random variable, and hence, as $\Delta t \to 0$,

$$\Phi(J_t) \xrightarrow{D} U(0,1),$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution and $U(0,1)$ denotes a uniform random variable.

Theorem 2.2 states that if there is no jump for the stock price process, the distribution of statistic J_t converges to a standard normal distribution. Therefore, when we prepare a QQ-plot graph for the data set of J_t under the null hypothesis "there is no jump for the stock process on $(t_{i-1}, t_i]$", the graph is a straight line which has almost a 45% gradient. Moreover, we draw a confidence interval with a certain significance level (SL) in the QQ-plot graph. If there are some real data outside the confidence interval, then we determine that jumps are present. After that, we calculate function $b(t)$, which is called a "belief measure" in Lee and Hannig [8], and when $b(t) \geq 1-(\mathrm{SL})$, we determine that there is a jump at time t. Then, we exclude time periods that are determined as large jumps from Theorem 2.1. Finally, the residual data are considered to be small jumps in the Lévy process. For more details on the belief measure, we refer readers to Section 3.4 in Lee and Hannig [8].

3. **Empirical Results for Japanese Markets under the Financial Crisis.** This section presents some empirical results with respect to Japanese markets. Here, we use the tick data from Nikkei-NEEDS in our experimental study and analyze the Nikkei225 index and composition elements of Nikkei225 for individual stocks. More Precisely, we exclude stocks that had stock splits in 2008. Andersen, Bollerslev, Diebold and Ebens [3] also point out that when 5-min-interval data are used, we can cut off microstructure noise like that in bid-ask bounces. We unify the significance level to 1% for every statistical test discussed in this paper.

3.1. **Existence of jumps.** This section presents the results for the presence of jumps in the Japanese stock market in 2008. Figure 3.1 [1] is a table that lists the average numbers of large jumps and small jumps in each industry sector and the Nikkei 225 obtained with the LH method. We can not only find that there are large jumps from the results, but small jumps, which could not be determined by Barada et al. [4] who used the BNS, JO, and LM methods. Compared to individual stocks (from "a" to "ai" in Figure 3.1), the Nikkei225 has fewer jumps for both types since the Nikkei225 is an index of several stocks so that the value is considered to be a kind of averaged value of several individual stock prices and fluctuations are smaller than those in individual stocks.

Figure 2 plots fluctuations in the Nikkei225 in 2008 (solid line), its volatility (dotted line and $y2$-axis), and three kinds of time-points ($\times$, $\triangle$ and $*$) in jumps for the Nikkei225. The cross $\times$ denotes a detected point for all jumps obtained with the LH method, the triangle $\triangle$ denotes a point detected in a large jump with the LH method, and the asterisk $*$ denotes a point detected in a large jump with the LM method, which was studied by Barada et al. [4]. The Lehman shock occurred worldwide in September 2008 and we can see some jumps during its span with both methods (LH and LM). The wild movements still continue in October but we cannot observe that many jumps with the LM method or test for large jumps with the LH method. Here, we need to look at volatility values and these rapidly increase in October; consequently, we can conclude that the main element responsible for violent fluctuations in October is variations in volatility. However, note that the LH method discovers several small jump points in October. The LH method detects 32 large jumps and 22 small jumps in 2008 and we find 30 large jumps with the LM method. According to the results, the LH method obtains more jumps than the LM method, and we can constantly observe small jumps throughout the year from Figure 2.

3.2. **Distribution of jump sizes and frequencies.** We will next present some results with respect to jump sizes and their frequencies, viz., their average values, estimated distributions, and the p-values for the Kolmogorov-Smirnov test, which is used when we

[1]Type: Types of industry, No: Number of individual assets in the industry, SD: Standard deviation, a: Electronics, b: Chemicals, c: Machinery, d: Metals, e: Food, f: Banking, g: Automotive, h: Ceramics, i: Construction, j: Medical Drugs, k: Services, l: Trade Firms, m: Retail Trade, n: Trains and Buses, o: Fibers, p: Real Estate, q: Precision Equipment, r: Telecommunications, s: Iron and Steel, t: Electric Power, u: Paper and Pulp, v: Marine Transportation, w: Other Manufacturing, x: Securities, y: Gum, z: Insurance, aa: Land Transportation, ab: Shipbuilding, ac: Gas, ad: Storage, ae: Oil, af: Other Finances, ag: Fisheries, ah: Air Transportation, ai: Mining, aj: Nikkei225.

		Large		Small	
Type	No	Mean	SD	Mean	SD
a	30	54.3	19.140	30.8	20.124
b	16	50.4	9.414	23.8	7.299
c	14	46.9	19.375	33.8	21.241
d	12	36.5	14.523	30.2	13.545
e	10	58.9	9.710	37.3	9.737
f	9	47.7	14.989	27.6	9.476
g	9	51.0	16.186	34.9	20.594
h	8	45.3	9.782	27.0	4.950
i	8	42.0	10.296	24.3	6.685
j	8	58.9	7.928	23.5	4.822
k	7	49.9	8.202	29.9	8.132
l	7	40.7	9.632	24.4	5.778
m	7	49.6	11.287	27.6	7.980
n	7	42.7	13.541	36.1	30.097
o	6	61.8	9.990	49.5	37.353
p	5	40.6	7.940	24.2	4.707
q	5	49.0	6.164	24.2	5.600
r	5	58.8	14.579	39.0	21.799
s	4	45.3	6.379	21.8	6.495
t	3	42.3	3.399	31.3	4.497
u	3	53.3	15.691	28.0	3.742
v	3	40.3	8.260	18.3	1.886
w	3	61.0	10.424	37.3	7.760
x	3	39.0	5.888	23.3	6.128
y	2	51.5	19.500	24.0	1.000
z	2	39.0	13.000	19.0	1.000
aa	2	47.0	4.000	22.0	5.000
ab	2	48.0	9.000	30.0	1.000
ac	2	36.5	3.500	19.0	2.000
ad	1	44.0	0.000	27.0	0.000
ae	1	32.0	0.000	33.0	0.000
af	1	27.0	0.000	22.0	0.000
ag	1	68.0	0.000	34.0	0.000
ah	1	53.0	0.000	42.0	0.000
ai	1	47.0	0.000	40.0	0.000
aj	1	32.0	0.000	22.0	0.000

FIGURE 1. Numbers of jumps by LH

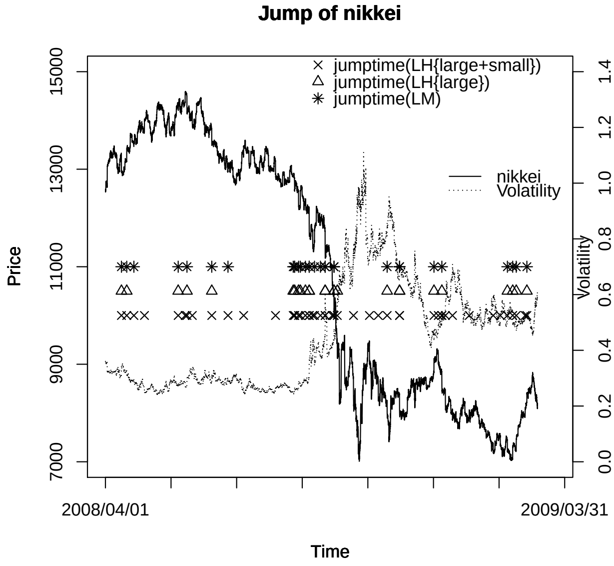

FIGURE 2. Nikkei225, volatility and jump-points by LH and LM methods

want to test whether two distributions are the same or not under the null hypothesis where both distributions are the same. We consider consistency in jump sizes and frequency distributions between an industry and the Nikkei225. To estimate the distributions we use the kernel density method with the Gaussian kernel (see [13]). And we also compared differences in distributions between 2008 and 2009.

The left-hand side of "Size-Large" and "Size-Small" columns in Table 1 summarize the average values for jump sizes, their right-hand side summarize the standard deviations (SD) in jump sizes, and the "Size-p-value" column summarize the p-values for the Kolmogorov-Smirnov test with respect to "Large" and "Small" jump-size distributions for all sectors of industry.

The average values for both types of jump sizes in almost all industry sectors and the Nikkei225 take negative values in 2008, and all of them are negative specifically in the large jump case. As we well know, because we had the financial crisis in 2008 and stock prices tumbled, the results are intuitive and can easily be accepted. However, as the p-values for jump-size distributions take various values, we are able to deduce various differences in jump-size distributions in sectors of industry. Note that the p-values for jump-size distributions in Trains and Buses (n), Telecommunications (r), Electric Power (t), Gas (ac) and Oil (ae) are larger than the others since they are stable stocks and fluctuations are close to those of the Nikkei225. We can also see that individual stocks are commonly considered to move more violently than the Nikkei225 index from the SD values in Table 1 (the SD for the Nikkei225 is the smallest). However, the SD values for these industry sectors are relatively small and therefore their jump-size distributions are close to those for the Nikkei225 index. In contrast, the p-values for Construction (i), Real Estate (p), Iron and Steel (s), Insurance (z) and Shipbuilding (ab) take small values, so that we can understand that they have different distributions from those in the Nikkei225 index for jump sizes and these are considered to be economically sensitive or business cyclical stocks.

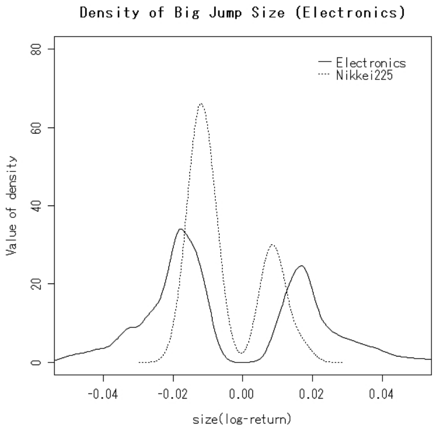

FIGURE 3. Electronics-size (large)

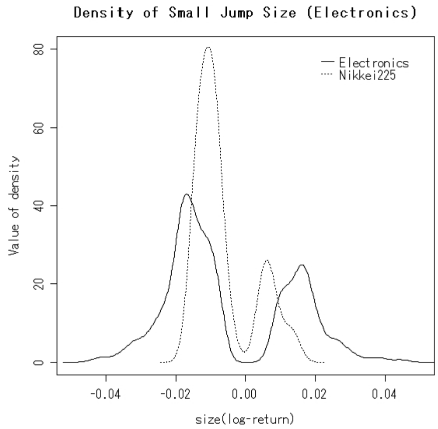

FIGURE 4. Electronics-size (small)

Figures 3 and 4 (solid line plots results for electronics sector (a) and dotted line plots the Nikkei225) give estimated distributions for both types of jump sizes in the electronics sector through kernel density estimation and we compare them to those of the Nikkei225. When we look at the p-values for electronics sector in the jump sizes of Table 1, we can see that they are comparatively small; thus, we see that the distributions are somehow different from those of the Nikkei225. We can also understand differences from the figures, especially for the features of the tails (Electronics sector has a fat tail). Therefore, when we model jump-size distributions for stocks in the electronics sector and other sectors that have small p-values in Table 1, we should use different distributions from those in the Nikkei225.

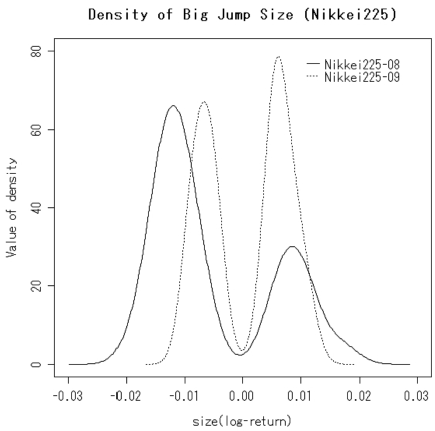

FIGURE 5. Nikkei225-size (large)

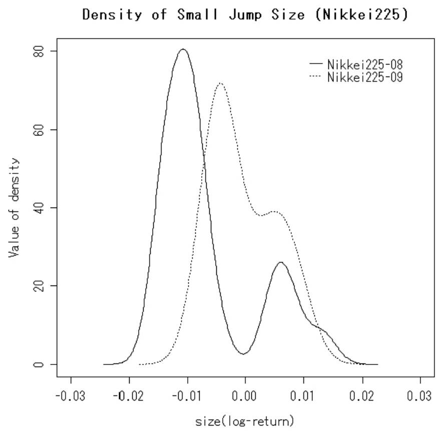

FIGURE 6. Nikkei225-size (small)

We can find differences between jump-size distributions for the Nikkei225 in 2008 and 2009 from Figures 5 and 6 (solid line denotes 2008 and dotted line denotes 2009). The mean values for "Large" and "Small" jump-sizes in the Nikkei225 in 2009 are 0.0007 for the former and -0.00048 for the latter and their p-values are 5.89×10^{-6} and 6.66×10^{-3}. Note that the mean values for "Large" jump sizes in 2009 takes a positive value and both p-values are quite small, so that fluctuations in jumps in 2008 markets can be distinguished from those in 2009. We can see that there are more and larger negative jumps in 2008 than in 2009 from the two figures.

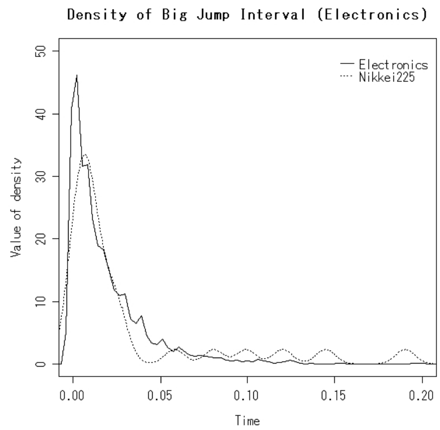

FIGURE 7. Electronics-freq. (large)

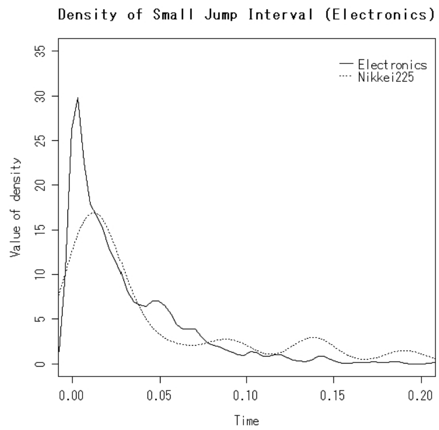

FIGURE 8. Electronics-freq. (small)

The right half of Table 1 provides average values for time intervals between one jump and the next (where "1" denotes one year so that "0.5" represents a half year), and SD values and p-values for frequency-distributions from the Kolmogorov-Smirnov test between industry sectors and the Nikkei225. We find from the p-values for frequency-distributions that most distributions of industry sectors do not differ from those of the Nikkei225. Hence, when we model jump frequencies, we do not distinguish individual stocks from the Nikkei225. We plotted graphs for the frequency distributions in the electronics sector (a) (Figures 7 and 8, solid line denotes Electronics and dotted line denotes the Nikkei225). It is difficult to find differences between distributions for the electronics sector and the Nikkei225 in either figures. An interesting finding from these figures is that the average value for large-jump frequencies in the Nikkei225 index is 0.0305, but the median value for large-jump frequencies in the Nikkei225 index is 0.0115. We can see from the average values that when there is a jump, the next jump occurs within 2 weeks, but from the median value, the probability that the next jump will occur within 3-days is 50%. Hence, we can understand that once there is a jump in the Nikkei225 index, we continuously have jumps in 2008. This can also be seen for individual stocks.

Finally, let us compare the frequency distributions for the Nikkei 225 in 2008 to those in 2009 as jump-size distributions (Figures 9 and 10, solid line denotes 2008 and dotted line denotes 2009). The average values for large and small jump time intervals correspond to 0.0279 and 0.1622 and the p-values for the Kolmogorov-Smirnov test for jump-frequency distributions are 0.1843 and 0.3647. We also cannot find differences in the years from the p-values.

4. **Conclusions.** We tested for the existence of Lévy jumps in Japanese stock markets with the Lee and Hannig method [8]. We specifically studied data from the Nikkei225 and individual stocks for 2008, in which the world financial crisis occurred. As a result, we observed not only large (Poisson) jumps but also small Lévy jumps. We also found the wild movements in September 2008 were caused by jumps, but the fluctuations in

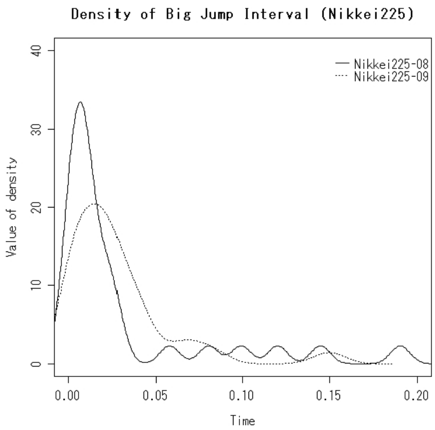

FIGURE 9. Nikkei225-freq. (large)

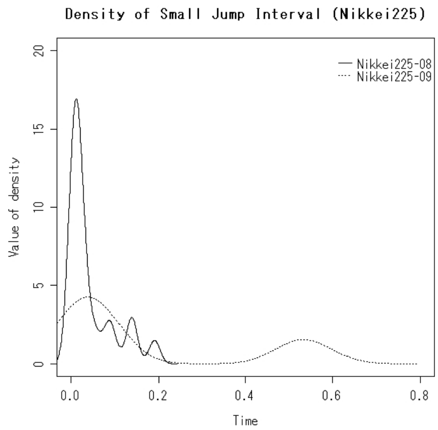

FIGURE 10. Nikkei225-freq. (small)

TABLE 1. Averages and p-values of jump-sizes and frequencies by LH method

		Size						Frequency					
		Large		Small		p-value		Large		Small		p-value	
Type	No	Mean	SD	Mean	SD	Large	Small	Mean	SD	Mean	SD	Large	Small
a	30	–0.0042	0.02479	–0.0052	0.01775	0.000270	0.003515	0.0179	0.02261	0.0298	0.03801	0.630299	0.620061
b	16	–0.0060	0.02179	–0.0053	0.01497	6.96×10^{-5}	0.034565	0.0189	0.02120	0.0401	0.04403	0.639243	0.372265
c	14	–0.0048	0.02624	–0.0051	0.01709	3.62×10^{-5}	0.018348	0.0207	0.02600	0.0273	0.03348	0.702508	0.443234
d	12	–0.0045	0.02520	–0.0031	0.01723	0.000260	0.010703	0.0252	0.02909	0.0320	0.03480	0.379381	0.662379
e	10	–0.0044	0.02004	–0.0059	0.01360	0.007263	0.079103	0.0164	0.01786	0.0254	0.02921	0.484602	0.480788
f	9	–0.0059	0.02452	–0.0045	0.01705	8.68×10^{-5}	0.040645	0.0205	0.02482	0.0327	0.03328	0.752800	0.830772
g	9	–0.0040	0.02329	–0.0034	0.01620	0.002313	0.055422	0.0192	0.02112	0.0271	0.03265	0.628966	0.484705
h	8	–0.0063	0.02559	–0.0048	0.01963	3.29×10^{-5}	0.010953	0.0214	0.02474	0.0352	0.03723	0.824356	0.590468
i	8	–0.0074	0.02213	–0.0063	0.01580	3.52×10^{-6}	0.013487	0.0231	0.03015	0.0391	0.04214	0.795954	0.704621
j	8	–0.0056	0.01820	–0.0047	0.01346	0.006038	0.321434	0.0165	0.01939	0.0406	0.04167	0.584457	0.459942
k	7	–0.0069	0.02647	–0.0066	0.01710	0.000734	0.042536	0.0190	0.01976	0.0315	0.03268	0.548142	0.792323
l	7	–0.0050	0.02229	–0.0028	0.01769	0.000583	0.011412	0.0241	0.02868	0.0376	0.04151	0.535678	0.889548
m	7	–0.0039	0.02326	–0.0042	0.01602	0.000515	0.037555	0.0194	0.02298	0.0348	0.03812	0.574834	0.811481
n	7	–0.0045	0.01448	–0.0025	0.01052	0.778617	0.189965	0.0222	0.02777	0.0271	0.04145	0.839361	0.143977
o	6	–0.0068	0.02224	–0.0066	0.01370	8.00×10^{-5}	0.021097	0.0053	0.11557	0.0065	0.12761	0.407823	0.069582
p	5	–0.0043	0.02740	–0.0078	0.01737	9.33×10^{-6}	1.10×10^{-5}	0.0234	0.02650	0.0397	0.03891	0.719512	0.390171
q	5	–0.0058	0.02158	–0.0031	0.01666	0.001860	0.045155	0.0194	0.02235	0.0383	0.03874	0.650859	0.298293
r	5	–0.0036	0.02010	–0.0008	0.01368	0.035086	0.006922	0.0164	0.02111	0.0237	0.03183	0.507675	0.182270
s	4	–0.0090	0.02117	–0.0069	0.01574	9.40×10^{-7}	0.009353	0.0217	0.02438	0.0413	0.05036	0.665415	0.938709
t	3	–0.0046	0.01439	0.0004	0.01239	0.197173	0.006406	0.0233	0.02808	0.0296	0.02822	0.538913	0.692615
u	3	–0.0064	0.02278	–0.0072	0.01467	0.000393	0.091131	0.0180	0.02284	0.0363	0.03203	0.726124	0.443901
v	3	–0.0036	0.02134	–0.0039	0.01715	0.000428	0.083371	0.0234	0.02771	0.0518	0.04867	0.436041	0.033994
w	3	–0.0074	0.01729	–0.0038	0.01433	0.007555	0.098011	0.0162	0.02302	0.0244	0.02677	0.474197	0.472799
x	3	–0.0034	0.02267	–0.0032	0.01923	0.001811	0.006391	0.0250	0.02383	0.0378	0.03442	0.235566	0.360956
y	2	–0.0042	0.02091	0.0002	0.01687	0.003024	0.011676	0.0183	0.02313	0.0414	0.04007	0.708681	0.659002
z	2	–0.0091	0.02298	0.0035	0.02006	2.83×10^{-5}	0.000176	0.0244	0.02576	0.0489	0.04662	0.392400	0.273074
aa	2	–0.0067	0.01684	–0.0033	0.01455	0.005320	0.145232	0.0101	0.01169	0.0227	0.02774	0.148167	0.334593
ab	2	–0.0081	0.02484	–0.0103	0.01719	3.27×10^{-5}	0.000551	0.0199	0.02393	0.0321	0.03406	0.750691	0.769867
ac	2	–0.0044	0.01656	–0.0067	0.01186	0.209296	0.415099	0.0257	0.03117	0.0518	0.04984	0.254101	0.229058
ad	1	–0.0085	0.02062	–0.0059	0.01712	0.004568	0.025461	0.0218	0.02210	0.0364	0.03779	0.861929	0.571618
ae	1	–0.0056	0.01646	–0.0016	0.01318	0.009516	0.138081	0.0263	0.03557	0.0276	0.03321	0.607005	0.518319
af	1	–0.0117	0.03842	–0.0060	0.02120	1.47×10^{-5}	0.049346	0.0369	0.03359	0.0459	0.03445	0.220454	0.364979
ag	1	–0.0029	0.02228	–0.0007	0.01557	0.009356	0.010352	0.0146	0.01784	0.0291	0.03491	0.511802	0.813594
ah	1	–0.0034	0.01235	–0.0007	0.00956	0.066787	0.012298	0.0185	0.02076	0.0218	0.02559	0.828017	0.171556
ai	1	–0.0029	0.02318	–0.0069	0.01275	0.000955	0.099722	0.0210	0.02587	0.0240	0.02813	0.798462	0.383508
aj	1	–0.0053	0.01035	–0.0066	0.00823	1	1	0.0305	0.04634	0.0435	0.05293	1	1

October were generated by rising volatility. Next, we compared the distributions in jump sizes for the Nikkei225 to those in industry sectors with the Kolmogorov-Smirnov test, and concluded that there were some differences in most sectors. We also found that there were differences between distributions for 2008 and 2009. However, when we compared distributions in the jump-time intervals between them, we could not detect differences in most industry sectors or years.

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